QP CODE: 23505503

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Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, APRIL 2023

First Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

Complementary - IPH1CM05 - MATHEMATICS - I DIFFERENTIAL CALCULUS, MATRICES AND TRIGONOMETRY

2021 Admission Onwards

4C309D5B

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the slope of the curve $x^3 + y^3 - 9xy = 0$ through the point $(2, 4)$.
2. State Mean value theorem.
3. Sketch the domain of the function $f(x, y) = \sqrt{y - x^2}$.Determine if the domain is an open region, a closed region or neither.
4. Draw the tree diagram for the function $z = f(u, v, w), u = g(t), v = h(t), w = k(t)$ and also write the chain rule.
5. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2,\ x = r - s,\ y = r + s$.
6. Define transpose of a matrix with an example.
7. Define non homogeneous system of equations.
8. Prove that $[\sin(\alpha + \theta) - e^{\alpha i} \sin\theta]^n = e^{-n\theta i} \sin^n \alpha$.
9. If x is real, show that $\sinh^{-1} x = log[x + \sqrt{x^2 + 1}]$.
10. Separate into real and imaginary parts the expression $\sin(\alpha + i\beta)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2y + xy^2 = 6$.

12. Find the absolute maximum and absolute minimum values of $f(x) = 10x(2 - ln\ x)$ on the interval $[1, e^2]$.

13. Find all the second order partial derivatives of $f(x, y) = x \cos y + ye^x$.

14. a. Draw the tree diagram for the function $w = f(x, y), x = g(t), y = h(t)$.
    b. Use the chain rule to evaluate $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$ if $w = xy, x = \cos t, y = \sin t$.

15. Solve $x_1 - 2x_2 + 3x_3 = 0$
    $2x_1 + 5x_2 + 6x_3 = 0$

16. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, find the inverse of A using Cayley Hamilton theorem.

17. Sum the series $\sin x + \frac{1}{2}\sin 3x + \frac{1.3}{2.4}\sin 5x + \ldots$ to ∞.

18. Sum of the series $1 + \cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots$ to ∞.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.
    b. Find the value of $\frac{\partial z}{\partial x}$ at the point $(1, 1, 1)$ if the equation $xy + z^3x - 2yz = 0$ defines z as a function of x and y.
    c. Use the limit definition of partial derivative to compute the partial derivatives of $f(x, y) = 4 + 2x - 3y - xy^2$ at the point $(-2, 1)$.

20. a. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$, then show that $\rho(AB) = \rho(BA)$.
    b. Find the rank of $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.

21. Find the eigen values and the corresponding eigen vectors of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

22. a. Prove that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$.
    b. Expand $\cos^7\theta$ in a series of cosines of multiples of θ.

(2×5=10 weightage)